Ockham's Razor[*]

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Entities must not be multiplied without necessity. How many times have we heard that ? It is one of those philosophical dogmas that get passed on from mind to mind without anyone really taking stock of them anymore. I have felt uneasy about Ockham's razor for a long time now, and I think that it has done a lot of damage, both philosophical and psychological.

It is interesting that philosophers such as Quine and Goodman, who appeal to Ockham's razor all the time, have not really given us a sustained justification for it. Most of what you get in the way of justification is philosophical gut belief. You are browbeaten into accepting Ockham's razor at the cradle; how could you not ?

So, what is the justification for Ockham's razor ? It is obvious that we are dealing with an epistemological principle, that among other things can be applied to ontology, and not with a specifically ontological principle. Its justification is basically the following. Given that my justifications are generally quite partial and that I am prone to error, I should be careful about what I assume to be the case, and, therefore, I should try to assume as little as possible. For the case of ontology, this *non-sequitur* means that I should try to keep my ontology down, *i.e.* not assume any unnecessary entities. One could embelish this, but it is the essential core of the argument.[1]

There is a big problem however, if you accept the argument, and it is this : How do you know which entities are necessary and which unnecessary ? You

certainly don't know it *a priori*, do you ? So, how do you proceed ? Well, one could start with certain philosophical intuitions, or prejudices, or tastes, and go on from there. You see what you can do directly, given your assumed basis, and what you can't do directly you try to get somehow. Sometimes this involves broadening your basis a bit, sometimes it involves some pretty fancy detours from the straight path. This is what Goodman does, most of the time, with a lot of very subtle work and some snide remarks about the opposition. This is also what Goodman and Quine started out doing in « Steps Toward a Constructive Nominalism ». Let's recall their initial declaration of principles :

> We do not believe in abstract entities. No one supposes that abstract entities
> — classes, relations, properties, *etc.* — exist in space-time; but we mean more
> than this. We renounce them altogether. [...] Why do we refuse to admit the
> abstract entities that mathematics needs ? Fundamentally this refusal is based
> on a philosophical intuition that cannot be justified by appeal to anything
> more ultimate. It is fortified however, by certain *a posteriori* considerations.
> What seems to be the most natural principle for abstracting classes and
> properties leads to paradoxes. Escape from the paradoxes can apparently be
> effected only by recourse to alternative rules whose artificiality and arbitrari-
> ness arouse suspicion that we are lost in a world of make-believe.[2]

Goodman has held fast to his basic philosophical intuition, and has limited himself to methods that he calls nominalistic or hyper-extensionalistic. It is interesting that his dislike of platonism led him to a version of nominalism that is, in a very clear sense, essentially negative. « Nominalism for me consists specifically in the refusal to recognize classes », he says.[3] This creates some problems because you could throw in anything you want into the world of individuals as long as you don't treat it as a class. « Granted. Nominalism no more guarantees philosophical soundness than the refusal to eat poison guarantees physical well-being. »[4] This is the point for Goodman; platonism is bad, poisonous claptrappery, whereas « nominalism is a restrictive rule of processing that won't select our raw materials or help us make good things out of bad materials but will keep us from making bad things out of good materials. »[5] It is a fight between Good and Evil, and « nominalism is good. »[6]

These issues always seem to be immersed in clouds of emotion, and any discussion, personal or in print, easily gets heavy with sarcasm and inuendo. We see Goodman, who is an otherwise well mannered and charming writer, getting on his armor and heading for the holy war at the slightest provocation, real or imagined. His positive nominalistic work is often very interesting and, occasionally, he even allows himself some (allegedly dispensable) platonistic methods; but the platonist really gets his goat.[7]

Quine has since recanted his basic philosophical intuition; and was duly chastened by Goodman — in good humor, of course.[8] What Quine realized is that he needed a broader approach; instead of starting from the bottom up,

one should start from the middle down (and maybe also a little up, but only when necessary).[9] This was probably a consequence of his rebellion against logical positivism and of his going back, armed with the logistic method, to some of the old questions of philosophy, such as the questions of ontology, essence, necessity, *etc*. This led him to the crucial argument that if we accept science, then we must be platonists. Why ? Because mathematics is an integral part of science, and mathematics is ontologically committed to abstract entities; hence, to platonism[10]. Abstract entities are necessary, so we must accept, or postulate, their existence

Actually, this is also a version of Ockham's razor, closer to what Boehner states as the true version :

> We are not allowed to affirm a statement to betrueor to maintain that a certain thing exists, unless we are forced to do so either by its self-evidence or by revelation or by experience or by a logical deduction from either a revealed truth or a proposition verified by observation.[11]

Quine is arguing that science, based on experience and deduction, among other things, does so force us to maintain the existence of abstract entities. Evidently, Goodman can reply that although science *seems* to force us to this conclusion, Quine really hasn't got a knock-down argument for the impossibility of a nominalistic reconstruction of science. It is a matter of the force of the forcing — or the compelling, or the needing.[12]

And, in fact, Quine did not really give up his basic nominalistic convictions; that's why he became a somewhat tortured platonist; or, better, a reluctant one. This reluctance is evidenced by his formal approach to platonism. He has no feel for it; it is mostly a cross to bear. He embraces platonism with his fingers crossed behind his back in the hope that some day Ockham's razor may cut things down again. This formal platonism is conceptualized by Quine in his theory of defective nouns. He starts with « sake » and ends up with « number »[13]. To see how unsatisfactory this theory really is, I will discuss his paradigm example; the so-called defective noun « ordered pair ».

Quine begins by finding fault with Peirce's definition of ordered pair, which, leaving aside some mentalistic connotations, characterizes an ordered pair as a pair that has associated with one of its members the notion of first, and with the other the notion of second[14]. Quine finds this unclear and claims that a pair is anything that satisfies the formal condition :

(OP) If $<x,y> = <z,w>$, then $x = z$ and $y = w$.

All definitions of ordered pair that satisfy this condition are right, even though they may conflict with one another. But that's the way of defective nouns; their reference is just filler for the cake you are eating. This means, for Quine, that

there is no underlying essence to ordered pairs, *i.e.* that there is nothing common to all the definitions *except* satisfying the formal condition (OP)[15]. Well, let's see.

Peirce's idea can be expressed as

(P) $<x,y>$ = ([x,First], [y,Second]),

where « First » and « Second » are basically labels attached to x and y, respectively, the brackets represent some sort of association, and the parentheses represent ordinary pairing — not necessarily set-theoretical. Suppose now that you are packing two different kinds of pills into little envelopes that get placed into a box. In each box there are two envelopes each containing one pill; one of one kind and one of the other. Whoever takes the pills is supposed to take them in a certain order. It may occur to you to put a little sticker on each envelope saying « First » and « Second », respectively. You soon realize however, that once you have tagged one of the envelopes there is no need to tag the other. (What else can it be ?) So, you decide to save on stickers and tag only one of them. Moreover, since the saving is considerable, you keep thinking of ways to economize. And in due time you come to realize that your tag doesn't have to be « First » (or « Second »); it can be anything you want as long as you've made up your mind whether you are tagging the first or the second member of the pair — and you inform the pill taker of that. It means the same thing. You call up the pill maker and ask him to make one of the kinds blue and the other one white, and you call up the box maker and ask him to write an instruction that the pill taker should take the blue pill first. You are then quite happy because you saved yourself a lot of money.

Hausdorff had a set-theoretic definition of ordered pair that was essentially like (P) but using the numbers 1 and 2 to tag x and y :

(H) $<x,y>$ = {{x,1},{y,2}}.

The only thing he had to make sure was that 1 and 2 were not in the range of the variables « x » and « y ».[16]

Wiener's definition of ordered pair[17] is a very natural saving on Hausdorff's, because it consists in tagging the first (or second) member of the pair using the empty set as tag :

(W) $<x,y>$ = {{x, $\varnothing$ },{y}}.

Moreover, Kuratowski's definition is just another variation on the same idea; the ultimate saving, since it consists in tagging the second (or first) member of the pair using the first (or second) member to produce a cardinality tag :

(K) $<x,y> = \{\{x\},\{x,y\}\}$.

This is the sort of idea that will only occur to mathematicians, who are used to working with abstract things and therefore do not see anything odd in taggings that may be physically odd. They are also used to elegance, and this tagging has the virtue of defining order from cardinality without bringing in any extraneous objects.[18]

In number theory you use the same idea (Peirce's) when you tag the first member of the pair with one number and the second with another, as in :

(N) $<x,y> = 2^x.3^y$.

The difference is that here you define ordered pair directly without appealing explicitly to a notion of pairing.

Peirce had the right idea. The *essence* of ordered pairs is to be *pairs that are ordered*. And if you have some notion of pair and you want to define ordered pairs, the way to do it is by some method of tagging that produces the ordering; any method that does it right, *i.e.* that allows you to distinguish the first from the second element of the pair, is a good method, though some may be better than others for a wide variety of reasons. The don't-cares are the accidental features and they are don't-cares precisely because, and only insofar as, they don't affect the essence of ordered pairs. But the pill distributor cared about saving money — and could save the money without qualms because he was preserving the essential feature of ordering the pills — and the mathematician cares about defining ordering from cardinality and about the elegance of achieving a pretty intrinsic ordering that does not appeal to extraneous objects. The condition (OP) is an abstract formulation of the essential feature of ordered pairs (being ordered), but it does not tell you anything about how to define ordered pairs in any particular theory. What Peirce does is to give you another formulation of the essential feature of ordered pairs, together with a good hint as to how to go about defining them in specific cases.[19]

So Quine's formalistic platonism is not convincing even for his paradigm case[20]. Quine was much too impressed by the paradoxes and thought that they destroyed any natural version of platonism. And when he was forced to accept platonism, by his own argument based on Ockham's razor, he could only treat it formally.

But, going back now to Quine's broader approach, he has a much better strategy for dealing with the problem I'm raising; that is, the problem of deciding which entities are necessary and which unnecessary. This strategy involves his notion of ontological reduction.[21] You develop various ontologies that can deal reasonably naturally with the problem at hand, and then you see if they can be reduced to more economic ontologies or to ontologies that, in some sense, are more acceptable to you. Unfortunately, Quine's formulation

of ontological reduction is not really very good; but the idea may be good.[22] It doesn't matter whether this idea can be formulated formally and generally, because all you need is to use it as a guiding principle. Thus Quine's work for dealing with meaning, with the mental, with propositional attitudes, with properties, with sets (virtual set theory, substitutional set theory).[23]

Is this the answer then ? Instead of starting with your tastes, and prejudices, and build, you take a broader attitude; you allow more, either because it's there, or because you are forced, and then you try to pare it down according to your tastes, and prejudices, and wit ?

It's not as simple as that. What do you have to do to show that certain entities are unnecessary ? Is it enough to show that in principle you could get rid of them ? What about practice ? (Could you show that you are a good catholic by showing that in principle you could go to mass every Sunday ?) One characteristic of these nominalistic reductions, once they start getting into essentially non-nominalistic domains, is that even if they were to succeed, they would only succeed in principle — and very much in principle.

Let me show you that in principle painting is unnecessary. We know from physics that, say, oil paints consist of various kinds of molecules. It may be possible in principle to construct a machine such that, for any given painting, it would place on a similar sized canvas (which it would also make with the same molecular structure) the right molecules in exactly the right places. Suppose that I can show this; a completeness theorem for the machine (program) relative to paintings. Therefore, painting is dispensable. What's wrong with this ? To begin with, the machine doesn't paint; it copies given paintings. No painting, no original, no copy. On its own the machine would be a total failure. But, couldn't we argue, if we had paintings produced by painting, then using the machine we could in principle get the same results, and, therefore, painting isn't necessary. Given the completeness theorem, we can « define » painting by describing the machine. It's pretty weak, isn't it ? But that's how the argument goes.[24]

Moreover, even if we could program the machine to « paint », would these be paintings ? (Since all paintings, past, present, and future, are presumably finite in number, in principle there is a program such that the machine would produce exactly those paintings.) Or, like the famous ape at the typewriter, in principle we could run the machine forever and, randomly, produce all paintings past, present, and future. Would we be producing paintings ? Could nature paint ? No, they wouldn't be paintings, for the same reason that parrots don't talk and apes don't write — until recently at least.

And the same holds for mathematics. The nominalist hasn't even begun to produce a mathematics-machine, even in principle. But even if he did, counterfactually, could his nominalistic mathematics *do* mathematics ? Could it have any meaning, any content, any truth, any beauty, any interest ? What is it good for ? It would be, at best, an ugly kind of parrot to be kept out of

sight muttering away. Only people who have no appreciation or use for mathematics would buy such machines.

The nominalist talks about mathematics as if all that matters is getting results.[25] The specific character of mathematical experience, knowledge, understanding, expression, *etc.*, does not concern him. It's all claptrappery, anyway. In fact, if mathematics weren't needed for science, he would be quite glad to throw it away. Well, painting isn't needed for science. Why don't you throw it away ? And it's not just a question of cognitive *versus* non-cognitive meaning, since we are all supposed to have left behind the dogma that what's cognitively meaningful is only what's analytic (in a language) or verifiable in experience. Maybe it's the taste for desert landscapes to which Quine often appeals.

This is not to say that properly nominalistic work is ugly or worthless. It can be very nice and very interesting as long as it doesn't get into the faking business. It has its place and it should stay there instead of trying to take over the world by dubious means.

It seems clear, then, that reduction in principle won't do the job. You may have some sort of theoretical result, that may actually be quite interesting as a result about the original notion, and yet if you try to pass it off for the original notion it becomes meaningless. And if you tried to work on your terms, the work would become impossible. You wouldn't understand. That's why a lot of these « reductions » are phony. You can see (or argue, or prove) how things *could* be done in those terms, but you *can't* do it that way. And it's no use talking about throwing away your ladder after you have climbed it, because what I'm saying is that you *can't* throw it away. It follows you forever as a crutch. In principle you've dispensed with something or other, but in practice you haven't, either practically or conceptually.

And, in fact, even reduction in practice may very well not do the job, for essentially the same reasons discussed above. A case in point is the case of music. This is a better analogy to mathematics in the sense that there is also a notational system, and a case where the notational system basically conforms to nominalistic standards.[26] Suppose now that somebody argued — Deafman, perhaps — that there is no music as such; that the only significance and intelligibility of music lies in the scores. (You think this is fanciful ? Go look at what Goodman and Quine are trying to do in their joint paper.)[27] But what about the *content* of music ? Is there no meaning, or emotion, or truth ? What about an awful performance of the score that cuts all that out and makes you feel like hiding under your seat ? It doesn't matter, says Deafman, as long as you follow the score it's the genuine article.[28]

Quine's complete proof procedures for first-order logic are in the same bag. It would do you no good to have a computer that could implement them, and spit things out at lightening speed in primitive notation, since your understanding of what is going on would be zilch. And they don't do the job

conceptually either, because *conceptually* that's not what logical truth is; just as conceptually painting is not placing molecules on a canvas and music is not writing scores.

This is a problem with reductions in general, whether ontological or not. What are the essential features of the stuff to be reduced that must be preserved by the stuff to which it is being reduced ? Usually, people are so much within the grasp of an idea, or an ideology, that they disregard the most blatant features of the original. They have some conception of essence that is very much distorted by the ideology and that wouldn't convince anyone who is not already in its grasp. Thus the idea that the only essential feature of mathematics is to be useful for science, and that the only essential feature of this usefulness is the formal expression of the results, and that the only essential feature of this formal expression is that it's done by means of symbols, and that the only essential feature of these symbols is syntactic, and that the only essential feature of syntax is given by the inscriptions that are used and by the rules for using them, and that the only essential feature of the inscriptions is their material constitution, and that the only essential feature of rules is to be algorithmic, and that the only essential feature of algorithms is to be something that in principle a Turing machine could do. You can cut it off at the first step, of course; but you can also cut it off at each subsequent step, even given the earlier steps. What's necessary and unnecessary, in a real sense, is not what appears in some philosopher's dream of desert landscapes, but what conforms with a multiplicity of conditions, both practical and theoretical, scientific, aesthetic, emotional, *etc.* Besides, this is not a dream; it's a nightmare.

Painting and music are part of reality, and it is not their usefulness or their indispensability for science, or for anything else, that makes them so. What the painter and the musician do is to explore reality; outer and inner. They express themselves through painting and music, and their paintings, performances, sketches, scores, notes, *etc.*, are the results of their explorations. What the mathematician does is to explore reality; outer and inner. He expresses himself through mathematics, and his theories, theorems, conjectures, speculations, guesses, *etc.*, are the results of his explorations.

As an absolute principle, which is what it purports to be, Ockham's razor is the expression of a philosophical castration complex. Let's be careful; don't stick your neck out; make sure first; go easy now; go only as far as you must. You want so badly your ship to stay afloat that, as long as it doesn't sink, you don't care whether it's sailing or being towed — forever.

What does have merit is Ockham's razor as a relative speculative principle. Even if your ontology is very strong, it may be important to examine carefully what can, and sometimes should, be done by means of certain limited principles. You may be able to kill flies with a machine gun, but a fly swater is much more effective — as all those dead people in Buñuel's *Le chien andalou* can testify.

But people are not content with this, and Ockham's razor's pernicious

influence has spread far beyond the issue of nominalism *versus* realism. *Everybody* feels obliged to use Ockham's razor, even the platonists. They feel that because they have an atom bomb they should use it to unclog the kitchen sink. They reduce up, so to speak. They feel that because they have one kind of abstract entity, then they *should* reduce all other kinds to that. They feel that since they have abstract mathematical objects, then they *should* demote mental constructions and abstractions, or formal constructions, or inscriptional constructions, to a *demi-mondaine* status of merely auxiliary stuff — they may be there, but they are denied any « serious » explanatory role. And this holds just as much for mentalists, and formalists, and materialists, and many other ists, because they are all in the grasp of Ockham's razor and feel that once they've settled on something as an important, or even essential, feature, then that's *all* there is to it — whatever the it may be.

And the necessity part of Ockham's razor, that allows you to go up when forced, is equally pernicious. People feel that they cannot just accept something because it's elegant, or satisfying, or by some other « frivolous » reason, but that they have to argue in terms of some solemn need, real or imagined. And if they cannot pull it through, they are willing to fold — even though they have a good hand and they are pretty sure that the other guy is bluffing. How many mathematicians have allowed themselves to be talked out of platonism by this philosopher's bluff ?[29] But some, like Hardy, called the bluff without appealing to necessity.[30]

And isn't it interesting that the « revelation » part of Ockham's razor is let slide into oblivion in the name of rationalism ? Any appeal to mysticism is supposed to go *against* Ockham's razor; you *should* use the razor to cut it down in favor of psychoanalytic or other explanations. The fact that a mystical experience can be totally *compelling* for the one who has it is not supposed to cut any ice. But if you read an account of western and eastern mystics, you'll see that it is not that easy to say that they are irrational in accepting the reality of their experiences.[31]

Instead of the strategy from the bottom up of Goodman's, and the strategy from the middle down and only forcibly up of Quine's, I would recommend a strategy from the top up and down. What is important is not that there shouldn't be more things dreamt up in your philosophy than there are in reality, but that there shouldn't be more to reality than is dreamt up in your philosophy. Start with as broad an ontology as you reasonably can; develop it and add to it as you see fit; pare it down, or change it somewhat, or expand it in some other way, or change it a lot, if you find an ugly or rotten part; look at parts of it in many different ways, but whenever you feel tempted to reject outright some of these parts, make sure that you are not simply bigoted — or be clear that you are.[32]

Many reductionist results are highly interesting, if for nothing else at least in the sense in which it is interesting that someone can sail across the Atlantic

on a surf-board. But crossing the Atlantic on a surf-board does not ontologically eliminate ocean liners, just as ocean liners do not ontologically eliminate surf-boards; if what you want to do is to catch waves, then you should use a surf-board rather than an ocean liner. And, as Thor Heyerdahl showed with the *Kon-Tiki* and the *Ra* expeditions, the possibility of going across the ocean on a very primitive craft can have important implications for human history and human culture. There are some interesting reductions here; real ones, both epistemological and ontological. He established a connection between Egypt, Peru, and Polynesia that can help explain many cultural similarities in accordance with the hypothesis of cultural diffusion.[33] But Heyerdahl didn't just sit around and argue that in principle it would have been possible to cross the Pacific, or the Atlantic, in such craft as that; he went out there and did it, because the evidence was very much against him.[34] People can still argue, of course, but at a different level.

Questions of ontology and epistemology are not simple yes/no questions, as Carnap and others have emphasized, nor questions to be settled by the one true scientific or philosophical methodology, as Feyerabend has argued. But it doesn't follow from this, as Carnap would want it, that therefore there are no good (epistemological) reasons to choose. Nor does it follow that less is beautiful, simply by being less, or that only those who want more should have to pay dues. There are many, many good reasons to choose; and among them, other things not dipping the scales too far down, aesthetic reasons may be some of the better ones — as theoretical scientists and mathematicians have emphasized over and over again. What is simplicity, after all ?

Plato's conception *is* beautiful, grandiose, awe-inspiring. Who can read the parable of the cave and not feel moved ? And why should you dismiss it so ? Just because it takes a long time to work out the details and fill up the holes ? Why is it that scientists and mathematicians can be so patient, and philosophers seem so impatient ? It has taken more than three thousand years to understand the circle and the sphere, and they are still working at it.[35] And yet, we, philosophers, are prepared to give up a theory about reality as a whole at the drop of a hat, because of some flimsy arguments and because there are holes to be filled up. How unreasonable can one be ? Of course, Plato's conception is not the only one that has aesthetic and other merits. The desert can be beautiful and tremendously varied, and awe-inspiring as well. (Remember those splendid shots in *Lawrence of Arabia* ?) But don't rationalize your legitimate taste for the desert by means of silly little absolute principles like Ockham's razor.

But, you may question, wouldn't this view of yours lead to a free for all ? Wouldn't it follow that anything goes ? Not at all. I'm working here within two basic assumptions of western philosophy; namely, that reality is rationally intelligible, and, therefore, that claims to knowledge and theoretical belief are to be, to a greater or lesser extent, rationally justified. The only version of

Ockham's razor that I would accept is one that compels me to stay within the limitations imposed by these assumptions, *i.e.*, in the case of ontology, that we should avoid in our philosophy entities and principles whose acceptance is irrational. But, unless you are very narrow-minded about what's rational, this principle gives you a very wide berth.[36]

Ockham's razor gained a lot of popularity with logical positivism because of the generalized illusion that Russell had shown that mathematics and logic are nominalistically reducible to open sentences and the other stuff at the bottom of the hierarchy; an illusion, or delusion, that was definitely blown by (among others) Quine himself in « Russell's Ontological Development ».[37] The hope was then shifted to formalism, and, in spite of Gödel's incompleteness theorem and other infirmities of formalism, that hope is still alive and kicking.[38]

Quine once said that Ockham's razor dulled its edge on Plato's beard.[39] It's dull, all-right, but it wasn't exactly Plato's beard that did it.

Notes

1. Here is how Russell argues in « The Philosophy of Logical Atomism », pp. 221-222 :

> I have naturally a bias in favour of the theory of neutral monism because it exemplifies Occam's razor. I always wish to get on in philosophy with the smallest possible apparatus, partly because it diminishes the risk or error, because it is not necessary to deny the entities you do not assert, and therefore you run less risk of error the fewer entities you assume.

I think that it is an interesting observation of Russell's that you don't have to deny the entities that you don't assume, but I don't think that thereby you run less risk of error. In some cases, you run more risk, in fact, because you have to substitute very contrived constructions in place of more natural ones. And this has to do with the other part of Russell's justification :

> The other reason — perhaps a somewhat frivolous one — is that every diminution in the number of entities increases the amount of work for mathematical logic to do in building up things that looked like the entities you used to assume.

2. Pp. 173-174.

3. « A World of Individuals », p. 156. This is a change in the declaration of principles « from the vaguely general to the more specific » — instead of abstractness in general, classes in particular.

4. *Ibid.*, p. 165.

5. *Ibid.*, p. 165.

6. *Ibid.*, p. 155.

7. To see how extreme is Goodman's dislike of platonism, look at the following remarks
from « A World of Individuals ».

> A key principle [...] is that the nominalist rejects classes as incomprehensible, but
> may take anything whatever as an individual. Some misguided criticism would
> have been obviated had enough attention been paid to this statement; but I suspect
> that some of my critics feel that they do me a kindness by not taking it seriously.
> (Pp. 156-157.)

> Nominalism as I conceive it [...] does not involve excluding abstract entities, spirits,
> intimations of immortality, or anything of the sort. (P. 157.)

> Our platonist [...] also admits all classes of classes of classes of atoms, and so on
> *ad infinitum*, climbing up through an explosively expanding universe toward a
> prodigiously teeming Platonic Heaven. He gets all these extra entities [...] by a
> magical process that enables him to make two or more distinct entities from exactly
> the same entities. (Pp. 158-159.)

> Nominalism does not protect us from starting with ridiculous atoms. It does
> protect us from manufacturing gimcracks out of sound atoms by the popular
> devices of platonism. (P. 165.)

> The nominalist [...] is looking for a nominalistic translation of everything that
> seems to him worth saving. [...] When Wang says : « So you see these occurrences
> of platonism are harmless after all », he completely discounts the fact that only the
> nominalist's efforts removed the sting. One might as well say that the program for
> eradicating smallpox in the United States is trivial because there is no more
> smallpox around. (P. 167.)

> The nominalist does not presume to restrict the scientist. The scientist may use
> platonistic class constructions, complex numbers, divination by inspection of
> entrails, or any claptrappery that he thinks may help him get the results he wants.
> (P. 168.)

It is interesting that if you don't discuss platonism emotionally, people don't take
you seriously. I once described a version of platonism to my friend Richard Epstein
over a couple of drinks in a bar in Campinas, and at the end he said : Well, that's very
interesting, but is it *platonism* ? (He was missing the Platonic Heaven, believed in with
the pit of the stomach, and seen with the eyes of the mind.)

8. « A World of Individuals », pp. 156, 170) :

> Quine has recently written that he would « now prefer to treat that sentence as a
> hypothetical statement of conditions for the construction in hand ». [...] and Quine
> agrees [with Carnap] that « the obvious counsel is tolerance and an experimental
> spirit ». Reluctant as I am to cast a shadow on all this sweetness and light, there
> are limits to my tolerance of tolerance. I admire the statesman tolerant of divergent
> political opinions, and the person tolerant of racial and educational differences;
> but I do not admire the accountant that is tolerant about his addition, the logician
> who is tolerant about his proofs, or the musician who is tolerant about his tone.

[...] Thus in place of Carnap's exhortation, I propose another : « *Let us, as philosophers, be utterly fastidious in choosing linguistic forms.* »

9. *Word and Object*, pp. 4-5 :

Analyse theory-building how we will, we all must start in the middle. Our conceptual firsts are middle-sized, middle-distanced objects, and our introduction to them and to everything comes midway in the cultural evolution of the race. In assimilating this cultural fare we are little more aware of a distinction between report and invention, substance and style, cues and conceptualization, than we are of a distinction between the proteins and the carbohydrates of our material intake. Retrospectively we may distinguish the components of theory-building, as we distinguish the proteins and carbohydrates while subsisting on them. We cannot strip away the conceptual trappings sentence by sentence and leave a description of the objective world; but we can investigate the world, and man as a part of it, and thus find out what cues he could have of what goes on around him. Substracting his cues from his world view, we get man's net contribution as the difference. This difference marks the extent of man's conceptual sovereignty — the domain within which he can revise theory while saving the data.

And in « On Multiplying Entities » (p. 264) he says :

We have to conclude that multiplication of entities can make a substantive contribution to theory. It does not always contribute. Of itself multiplication of entities should be seen as undesirable, comformably with Occam's razor, and should be required to pay its way. Pad the universe with classes and other supplements if that will get you a simpler, smoother overall theory; otherwise don't. Simplicity is the thing, and ontological economy is one aspect of it, to be averaged in with others. We may fairly expect that some padding of the universe is in the interest of the overall net simplicity of our system of the world.

10. Here is one version of the argument from « The Scope and Language of Science », p. 244 :

But we need to add *abstract* objects, if we are to accomodate science as currently constituted. Certain things we want to say in science compel us to admit into the range of values of the variables of quantification not only physical objects but also classes and relations of them; also numbers, functions, and other objects of pure mathematics.

This is, *mutatis mutandis*, « the » acceptable argument for platonism; we can't do without those entities. Even Plato used it in the *Parmenides* 135B-C :

[...] if [...] a man refuses to admit that Forms of things exist or to distinguish a definite Form in every case, he will have nothing on which to fix his thought, so long as he will not allow that each thing has a definite character which is always the same; and in so doing he will completely destroy the significance of all discourse.

(Not that Quine would accept this particular version of it.)

11. In his introduction to Ockham's *Philosophical Writings*, p. xx. The version of Ockham's

razor from which I started is not supposed to have been used by Ockham in exactly that form. The formulations he used which are closer to it are : « Plurality is not to be posited without necessity », and « What can be explained by the assumption of fewer things is vainly explained by the assumption of more things. » *Ibid.*, p. xxi. This is in agreement with Moody's account in *The Encyclopedia of Philosophy, Vol. 8* (« William of Ockham »), p. 307.

12. If scientists claimed that their scientific discoveries, and scientific methodology, depended on communication with spirits, divination by examination of entrails, or what not, he wouldn't agree that he is thereby forced simply by the efficiency of science to accept the existence of spirits or the soundness of the method of divination by entrails, however much ontological commitment there may be to these things. And, as we have seen, platonistic mathematics is at the same level for Goodman. I would certainly agree with Goodman on this point and stick to my guns rather than accept something that I consider to be totally incomprehensible, however necessary it may seem. One can keep working and take an attitude of wait and see. Why should the problem be solved *now* ? And Carnap's point about being prepared to shut up (in « Empiricism, Semantics, and Ontology », p. 23) is also a way of undercutting the argument by denying that we are compelled to accept the language of science (or the thing language, or whatever) — and, therefore that we are compelled to accept the conclusion of the argument. Of course, Quine argues that his attitude is the more *rational now*, but neither Goodman nor Carnap would agree.

13. *Word and Object*, chapter 7.

14. This is what Peirce says (*Collected Papers, vol. 2*, pp. 180-181):

> This Pair is a single individual object having this relation to Cain and to Abel, that its existence *consists* in the existence of Cain and in the existence of Abel and in nothing more. The Pair, though its existence thus depends on Cain's existence and on Abel's, is, nevertheless, just as truly existent as they severely are. The *Dyad* is not precisely the Pair. The Dyad is a mental diagram consisting of two images of two objects, one existentially connected with one member of the Pair, the other with the other; the one having attached to it, as representing it, a Symbol whose meaning is « First », and the other a Symbol whose meaning is « Second ». Thus, this diagram, the Dyad, represents Indices of Cain and Abel, respectively....

15. *Word and Object*, pp. 258-259 :

> This construction is paradigmatic of what we are most typically up to when in a philosophical spirit we offer an « analysis » of some hitherto inadequately formulated « idea » or expression. We do not claim synonymy. We do not claim to make clear and explicit what the users of the unclear expression had unconsciously in mind all along. We do not expose hidden meanings, as the words « analysis » and « explication » would suggest; we supply lacks. We fix on the particular functions of the unclear expression that make it worth troubling about, and then devise a substitute, clear and couched in terms to our liking, that fills those functions. Beyond those conditions of partial agreement, dictated by our interests and purposes, any traits of the explicans come under the head of « don't-cares ». Under

this head we are free to allow the explicans all manner of novel connotations never
associated with the explicandum. This point is strikingly illustrated by Wiener's
[definition].

No doubt Quine would also object specifically to Peirce's mental imagery, but as can
be seen from the above remarks it is not the mentalism *per se* that he finds objectionable.

16. See *Grundzüge der Mengenlehre*, p. 32.

17. In « A Simplification of the Logic of Relations », p. 225.

18. Kuratowski « Sur la notion d'ordre dans la théorie des ensembles ». The pill
manufacturer can do the same thing by placing a blue pill in one of the envelopes and
a blue and a white pill in the other, and adding the instruction : take the pill in the
envelope containing only one pill first and the pill of a different color than it that's in
the envelope containing two pills second — and throw out the remaining pill. This
would be considered to be odd, and wasteful, but it could even make good sense under
special circumstances. It would be a good solution, for example, if he only had the pills
(of different colors), the envelopes, and the instruction to work with, and was stuck
with pill takers who could count but who could only tell the *relative difference* between
those colors. In fact, in this physical case you can save yourself the envelopes and the
counting by saying : Take one of the pills of the same color first and the one of a different
color second. This is somewhat like the case of number-theory (coming up in the text)
because given the three pills there is a unique decomposition into two pills of the same
color and one pill of a different color. (There are many variations one can make on this
sort of example.)
    After presenting these definitions, and some for number theory as well, Quine
comments (*Word and Object*, p. 260) :

> Which is right ? All are; all fulfil [(OP)], and conflict with one another only out
> among the don't-cares. Any air of paradox comes only of supposing that there is
> a unique right analysis — a mistake that is encouraged by the practice, otherwise
> convenient, of using the term « ordered pair » for each version.

What led me originally to motivate Kuratowski's definition in this way was not
Quine's view, or Peirce's, but student's reaction to the definition. When the definition
is unmotivated, as in Mates' *Elementary Logic* and all other introductory texts that I have
used in teaching, the tendency of students is to be puzzled and to reject it. Even if one
carefully proves (OP) and illustrates how to recognize the first and second elements of
the pair, the most one gets is a reluctant sort of acceptance. The explanation in terms
of tagging, on the other hand, is easy to get accross because students immediately see
the *idea*. And they think it's neat that *it* can be implemented in so many different ways,
because it shows them something about mathematics — and not only about
mathematics.

19. Quine also suggests that « mathematicians pretty deliberately introduced [the
notion of ordered pair], subject in effect to the single postulate [(OP)] » (*Word and Object*,

p. 258). I think that this is rather misleading. To begin with, (OP) *goes without saying*. What else could an ordered pair be but a pair whose components are ordered and hence something that satisfies (OP) ? As far as I can see, the issue of whether ordered pairs satisfy (OP) didn't come up; it was simply assumed. Whitehead and Russell, for example, do not formulate (OP), but when first discussing ordered pairs they remark : « Such a couple has a *sense, i.e.* the couple (x,y) is different from the couple (y,x), unless $x=y$. We shall call it a « couple with sense, » to distinguish it from the class consisting of x and y. It may also be called an *ordered* couple. » *Principia Mathematica, Vol. 1*, p. 26n. (OP) expresses this obvious feature of ordered pairs and can be said to characterize this notion in the sense that any adequate definition will turn out to satisfy (OP).

20. It is worse when he gets to numbers because, due to his rejection of second-order logic, he cannot even formulate formally the counterpart of (OP) for natural numbers. Instead he talks (informally) of progressions (*Word and Object*, p. 258) :

> The conditions upon all acceptable explications of number (that is, of the natural numbers 0, 1, 2, ...) can be put almost as succintly as [(OP)] : any *progression* — *i.e., any infinite series each of whose members has only finitely many precursors — will do nicely.*

But this won't do, because if he appeals to a first-order formulation of arithmetic to write down his condition, then he won't get just the progressions; and he can't appeal to sets, either in an absolute sense — because « set », like « sake », « ordered pair » and « number », should be a defective noun for him, — or in the sense of a first-order set theory, because, for the same reason, he cannot fix the interpretation of this theory as *sets* rather than, say, numbers. And even disregarding this, the view of numbers with which he ends up has all the shortcommings of his view of ordered pairs (*Ibid.*, p. 263) :

> Frege's progression, von Neumann's, and Zermelo's are three progressions of classes, all present in our universe of values of variables (if we accept a usual theory of classes), and available for selective use as convenient.

21. See « Ontological Reduction and the World of Numbers ».

22. At least for the case of mathematics, Quine's criterion of ontological reduction, in terms of proxy functions that preserve structure, is really a criterion of ontological commitment. If you show that the structure of numbers is present in the ontology of sets, then what you have really shown is that set theory is ontologically committed to numbers. This is if you accept that the essence of mathematics is structure, as the old saying goes, and not some particular mathematical or non-mathematical content of one or another realization of that structure. This is also what's wrong with Quine's theory of defective nouns. It *seems* to be a move in the direction of recognizing structure as the essential thing, but then Quine sticks to the realizations and claims that there is no common underlying structure. The only sense in which we can talk of reduction in connection with Quine's criterion is epistemological, not ontological; you don't need to postulate the structure of numbers *in addition* to the structure of sets because the *structure* of numbers is *already there*.

23. I remember well Quine's excitment when he presented a preliminary version of *The Roots of Reference* at Irvine in 1971. He thought that he could do set theory substitutionally and effect a pretty grand reduction. Nobody was really convinced, and throughout the six weeks there were many attempts to find a hole. Finally, during the last lecture, just as Quine was about to catch his plane, Thomason asked him something about the axiom of choice and Quine proceeded to show that it was false in his theory — if I remember correctly, essentially by Russell's argument that there is no rule for selecting one each from an infinite collection of pairs of socks. This made the hole clear to me and I asked Quine : How do you say in your notation that there is a singleton whose only element is not a number ? — the last qualification was necessary because Quine had natural numbers in the system. Quine has since called this « The Law of Unit Subclasses », and I earned myself a footnote. Well, naturally, at the time I was thrilled that I had found the hole in Quine's reduction. But now I see it differently. Since there was a hole, there was nothing to be done, and Quine took it in good stride; but it was a pity, because it may have been an interesting result and, as I argue below, it wouldn't have taken anything away from the platonist anyway.

24. This is how Quine argues that one can eliminate the notions of truth and validity for first-order logic in terms of purely syntactical proof procedures (*Philosophy of Logic*, p. 57) :

> The key to these new definitions is the completeness theorem, above. We can simply define the moves that constitutes one of these complete proof procedures, and then define a valid schema as a schema that can be proved by such moves. Then we can define a logical truth derivatively as before : as a sentence obtainable by substituting for the simple schemata in a valid schema. Actually some of those complete methods of proof do not require schemata, but can be applied outright rather to the sentences that would be the results of substitution in the schemata. Such a method serves to generate logically true sentences directly from other logically true sentences. If we choose one of these proof procedures, we can skip schemata and validity; we can simply define a logical truth as any sentence produced by these rules of proof. Any such proposal, to define validity or logical truth in terms of proof procedure, tends to call forth a clamor of protest. It is protested that the property of being provable by the chosen proof procedure is intrinsically uninteresting; it derives its interest solely from the completeness theorem, which equates it with logical truth in a prior and intrinsically interesting sense. It is protested also that in so defining logical truth we would pull the rug from under the important completeness theorem itself, depriving it of content. Actually, no such matters are at stake. The completeness theorem as formulated [in terms of satisfaction in models] is independent of how we define logical truth, since it does not mention logical truth by name. Part of its importance is that it shows that we can define logical truth by mere description of a proof procedure, without loss of any of the traits that may have made logical truth interesting to us in the first place.

If this is not a case of wanting to eat your cake and have it too, I don't know what is.

25. It is interesting that Goodman complains against the pragmatist's « Law of Getting Results » without realizing (or acknowledging) that such nominalistic reductions as he proposes for mathematics deal only with results. At least in the case of painting, the

paintings are the results, and the painting-machine is perfect at reproducing them —
one does buy reproductions, after all, and hang them up. But the mathematics-machine
would give you reproductions that are so distorted that nobody would ever dream of
hanging them up. See « A World of Individuals », p. 170.

26. See Goodman's *Languages of Art*, pp. 179-192. In p. 186, he says :

> I have been able to discuss here, rather sketchily, only a few salient samples of relevant
> questions concerning the standard language of musical scores. The results suggest,
> however, that it comes as near to meeting the theoretical requirements for notationality
> as might reasonably be expected of any traditional system in constant actual use, and
> that the excisions and revisions needed to correct any infractions are rather plain and
> local. After all, one hardly expects chemical purity outside the laboratory.

27. Goodman and Quine « Steps Toward a Constructive Nominalism », p. 182 :

> It may naturally be asked how, if we regard the sentences of mathematics merely
> as strings of marks without meaning, we can account for the fact that mathe-
> maticians can proceed with such remarkable agreement as to methods and results.
> Our answer is that such intelligibility as mathematics possesses derives from the
> syntactical or metamathematical rules governing those marks.

This goes much further than Hilbert's formalism because Hilbert, as a mathematician,
never questioned the *intelligibility* of non-finitistic mathematics in non-formal terms.
The formalist depends on the intelligibility of mathematics to produce his formalism.
This is already an attempted ontological reduction of the objects and structures of
mathematics to strings of formal symbols and rules for manipulating them. The
nominalist takes over where the formalist leaves off, working as if the formalist had
been successful in establishing that the only significant features of mathematics are
given by the formalism. Since, however, the formalism is already an idealization over
what can be constructed according to nominalistic methods, he castrates mathematics
even more by demanding that the formal strings be constructible in principle according
to his own idealization, that takes into account the amount of inscriptions which are
present in the universe, even if in a disconnected form. So, he is not concerned by what
is in practice meaningful and feasible, as the strict finitist is, but only by his own
restricted and restrictive philosophical notion of having the same content.

28. *Languages of Art*, p. 186 :

> Since compliance with the score is the only requirement for a genuine instance of
> a work, the most miserable performance without actual mistakes does count as
> such an instance, while the most brilliant performance with a single wrong note
> does not.

And now comes Goodman's *penchant* for formalism — in accordance with his principle
of linguistic intolerance. He continues (pp. 186-87) :

> Could we not bring our theoretical vocabulary into better agreement with common
> practice and common sense by allowing some limited degree of deviation in

> performances admitted as instances of a work ? [...] ordinary usage surely sanc-
> tions overlooking a few wrong notes. But this is one of those cases where ordinary
> usage gets us quickly into trouble. The innocent-seeming principle that performan-
> ces differing by just one note are instances of the same work risks the consequence
> — in view of the transitivity of identity — that all performances whatever are of
> the same work. If we allow the least deviation, all assurance of work-preservation
> and score-preservation is lost; for by a series of one-note errors of omission,
> addition, and modification, we can go all the way from Beethoven's *Fifth Symphony*
> to *Three Blind Mice*.

And so down the slippery slope of formalism we fall. It doesn't seem to bother
Goodman that what this may show is the inadequacy of the formalism thus literally
applied.

I am *not* accusing Goodman of lacking appreciation for art; on the contrary, I know
that he's got a great deal of it. And the points he is making in *Languages of Art* are *not*
designed to ontologically eliminate music. That's precisely why I called in his disciple
Deafman; one of those, holier than the Pope, who put 2 and 2 together and draw
conclusions. And I'm running Goodman together with Quine, in spite of their
differences — Deafman plays poker with Wyman and McX. Nevertheless, Deafman is
to music what Goodman is to mathematics.

29. In *Eléments d'histoire des mathématiques*, Bourbaki says (p. 30) :

> [...] and even in our days, many a mathematician who flaunts an intransigent
> formalism, would, deep inside, willingly subscribe to Hermite's opinion : « I
> believe that the numbers and functions of Analysis are not an arbitrary product of
> our mind; I think that they exist outside us with the same character of necessity
> as the things of objective reality, and that we meet them or discover them, and
> study them, just as the physicists, the chemists and the zoologists ».

(The reference to Hermite is to his correspondence with Stieltjes, Vol. 2, p. 398.) And
this is Bourbaki !

30. See « Mathematical Proof » and *A Mathematician's Apology*.

31. Here is a quote, from Scharfstein's *Mystical Experience* (p. 6), of St. Teresa of Avila's
experience of God's love appearing to her in the form of an angel :

> He was not tall but short, and very beautiful; and his face was so aflame that he
> appeared to be one of the highest ranks of angels, who seem to be all on fire. [...]
> In his hands I saw a great golden spear, and at the iron tip there appeared to be a
> point of fire. This he plunged into my heart several times so that it penetrated my
> entrails. When he pulled it out, I felt that he took them with it, and left me utterly
> consumed   the great love of God. The pain was so severe that it made me utter
> several moans. The sweetness caused   this intense pain is so extreme that one
> cannot possibly wish it to cease, nor is one's soul then content with anything but
> God. This is not a physical, but a spiritual pain, though the body has some share
> in it — even a considerable share. So gentle is this wooing which takes place
> between God and the soul that if anyone thinks that I am lying, I pray God, in His
> goodness, to grant him some experience of it.

(The quotation is from *The Life of Saint Teresa*, pp. 192, 210.)

Scharfstein remarks on the « transparently sexual imagery » of the description, and you may want to explain it as some sort of sublimation of sexual desire (he doesn't) and dismiss it by Ockham's razor — although even if there was a strong similarity to sexual experience, it doesn't *follow* that this was the *only* real aspect of the experience. But, in any case, if you can use Ockham's razor, why can't she ? And why would she be irrational ? After all, her experience was much more *compelling* to her than Quine's argument is to any of us, including Quine himself. But Quine has an *argument* ! So ? If Ockham thought that you could achieve that sort of thing by arguments, he would have been sinning against his own principle by adding an unnecessary clause. He is acknowledging that besides sensory experience there may be other kinds of experience that are equally compelling.

32. In *Against Method*, Feyerabend argues that bigotry, among other unholy things, is *essential* for the development of science. So, by Ockham's razor, you should keep it in your ontology. (But it may be essential for bigotry to work that you firmly believe that you aren't bigoted.) Goodman takes up the charge that nominalism is bigoted in « A World of Individuals », pp. 169-170. He agrees that one shouldn't be prejudiced and dogmatic, but one should be clear. « The nominalist shuns platonistic devices precisely because he feels that their use would defeat rather than serve the purpose of philosophy. A clear story cannot be told in unintelligible language. » His principles « are stipulated as prerequisites of soundness in a philosophical system. They are usually adopted because a philosopher's conscience gives him no choice in the matter. » Well, I can accept this. It may even be that one truly feels that platonistic language is unintelligible. But isn't it a little odd that almost everyone feels that it *is* intelligible, even though it may not be to their taste ? What kind of unintelligibility is this ? Wouldn't it be better to characterize nominalism positively, rather than negatively as an anti-set-theoricism ? If hyper-extensionalism is such a characterization, then why complain so much about the opposition ? And if you can't, what does this mean ? Is unintelligible set theory *essential* for your understanding of your own position ? How can you understand it, then ? This is the sense in which I find Goodman's nominalism bigoted; it characterizes itself as holy by rejection of what it sees as unholy. This has nothing to do, of course, with Goodman himself as an individual.

33. Heyerdahl relates (*The Kon-Tiki Expedition*, pp. 20-21) :

> So it had begun, by a fire on a South Sea island, where an old native sat telling legends and stories of his tribe. Many years later I sat with another old man, this time in a dark office on one of the upper floors of a big museum in New York. ...
> « No ! », he said. « Never ! »
> ...
> « You're wrong, absolutely wrong, » he repeated, and shook his head indignantly to drive out the idea.
> « But you haven't read my arguments yet, » I urged, nodding hopefully towards the manuscript which lay on the table.
> « Arguments ! » he said. « You can't treat ethnographic problems as a sort of detective mystery ! »
> ...

He pushed the unopened manuscript carefully to one side and leaned over the table.

« It's quite true that South America was the home of some of the most curious civilizations of antiquity, and that we know neither who they were nor were they vanished to when the Incas came into power. But one thing we do know for certain — that none of the peoples of South America got over to the islands in the Pacific. »

He looked at me searchingly, and continued :

« Do you know why ? The answer's simple enough. They couldn't get there. They had no boats ! »

« They had rafts, » I objected hesitatingly. « You know, balsa-wood rafts. »

The old man smiled and said quietly :

« Well, you can try a trip from Peru to the Pacific islands on a balsa-wood raft. »

And, referring to a list of « sixty specialized cultural parallels between ancient Peru and Egypt » produced by « one of the most zelous advocates of the theory of defending a total isolation of America before Columbus », he remarks (*The Ra Expeditions*, pp. 27-28) :

The list could provoke conclusions. It was in fact intended to do so. The author of the article concluded that, since Egypt is in eastern Africa and Peru in western America, there are two continents and a whole Atlantic Ocean between them. Two cultures both of which used reeds for boat building could not have had contact accross such distances, as a read boat cannot traverse an ocean. Accordingly, the sixty cultural parallels must have arisen independently of each other; they could not for practical reasons have been the result of a human voyage. The lesson to the reader : diffusionists [...] must stop nosing around after cultural parallels, because it is hereby demonstrated that such parallels prove nothing.

And in *Aku-Aku* (chapter 5) Heyerdahl tells how he got the mayor to show him the long-ears' secret of carving and raising the enourmous statues of Easter Island, thereby clearly undercutting claims of extraterrestrial origin. (The long-ears went proxy for the martians, as Quine would want it.)

34. *The Ra Expeditions*, p. 3 :

A papyrus reed is a soft, sappy flower stem which a child can bend and crush. When it is dry it snaps like a matchstick and burns like paper. On the ground in front of me lay a tinder-dry papyrus reed, savagely screwed and fractured into a zig-zag tangle. It had been thrown there in the morning by an indignant old Arab who mangled it between his fingers before flinging it away from him on the sand, spitting after it and pointing scornfully. « That thing, » he said, « that wouldn't even hold a nail; and how could you fix sails to a thing like that ? » The old man was a canny boat-builder who had taken the bus up from Port Said to conclude a contract for masts and rigging for the vessel we were building. He was so outraged that he took the next bus back to the coast.

35. I have a huge old book *A Treatise on the Circle and the Sphere*, by Coolidge, that attests well to this kind of painstaking work.

36. Of course, Ockham's razor is a version of the venerable principle of sufficient reason — see Boehner *Op. Cit.*, p. xxi — and so the issue comes down to differing conceptions

of rationality. But even some of the staunchest defenders of rationality admit that the acceptance of contradictory theories, refuted theories, theories with holes and unclarities in their foundations and development, etc., is often rational. And argument doesn't cut too much absolute ice either, because we are tired of knowing in philosophy that you can argue well for almost anything, however ridiculous it may seem. That science is the main monument of western rationality I have little doubt, but not in the sense of adherence to fixed methodological rules — as Feyerabend has shown in *Against| Method.* And it is quite unfair to portray Feyerabend as a defender of irrationalism because of his pinching of an overblown baloon. Whether one agrees or disagrees with his own methodological recommendations, and with his argued defence of them, he is well within the limits of western rationalism, and he concludes his book with the following words (Op. Cit., p. 309) :

> There is no need to fear that such a way of arranging society will lead to undesirable results. Science itself uses the method of ballot, discussion, vote, though without a clear grasp of its mechanism, and in a heavily biased way But the rationality of our beliefs will be considerably increased.

Staunch rationalists seem to have little confidence in rationality and think that unless they have their own absolute Ten Commandments all will be lost to the forces of obscurantism. And maybe they are right. But then what ?

37. After writing the first version of this paper I hit upon Hans Hahn's paper « Superfluous Entities, or Occam's Razor ». I started at the end and read, to my amazement, the poem with which he greets those who embrace the « transparently clear teaching of the world-affirming philosophy » (p. 19) :

> You emerge from death, uncertain sufferings,
> All that was faint and mean and vague,
> And learn with unobstructed vision to distinguish
> The sickly dawn from the clear day !

My God, I thought, he's defending Plato ! But, alas, Plato gets it in pages 4-5 (and Parmenides, and Aristotle; mystics one and all) :

> On the other hand, the philosophy that affirms the sensible world says that it is enough if there are horses to be found in the sensible world; it is superfluous to assume that there exists an idea « horse » besides them, an entity corresponding to the concept horse. *Sufficiunt singularia, et ita tales res universales omnino frustra ponuntur.* Or, in translation : « Individuals suffice, and so it is entirely superfluous to assume those universals. » And since they are superfluous — and this is the application of Occam's razor — away with them !

And in page 17 Russell comes in :

> It therefore seems that we cannot after all manage with the entities of the sensible world alone, but that we must assume, besides these entities of the first order, still other entities of a higher order which will withstand Occam's razor, namely classes of entities of the first order, further, classes of such classes (numbers were such

this point too the world-affirming philosophy has shown us the right way : Russell has shown how every meaningful statement in which the word « class » occurs can be transformed into a statement which mentions only objects themselves and which no longer mentions classes of objects.

38. And so Goodman's search for safe rules of processing and his principle of linguistic intolerance that glorifies, among other things, a very formalistic approach to philosophical problems characteristic of a lot of contemporary philosophy, from logical positivism onwards. This principle, it seems to me, has helped make a lot of bad things out of good materials; as we learn even from Goodman's own work.

An interesting case is his treatment of counterfactual conditionals in « The Problem of Counterfactual Conditionals ». His basic idea, a good one in my view, is that a counterfactual is true if the consequent follows by law from the antecedent together with true relevant conditions. Instead of reflecting upon what makes a condition relevant (or legitimate), however, he launches himself in the search for a formal criterion. After a few skirmishes he reaches the following (p. 13) :

> Our rule thus reads that a counterfactual [with antecedent A and consequent C] is true if and only if [a] there is some set S of true sentences such that S is compatible with C and with $\neg C$, and such that A & S is self-compatible and leads by law to C; while [b] there is no set S' compatible with C and with $\neg C$, and such that A & S' is self-compatible and leads by law to $\neg C$.

Now, the reason for the split clauses [a] and [b] is that for one Jones who is in neither Carolina, [a] lets through both counterfactuals « If Jones were in Carolina, he would be in North Carolina », using as S the fact that he isn't in South Carolina, and « If Jones were in Carolina, he would be in South Carolina », using as S the fact that he isn't in North Carolina. The information used as S is clearly illegitimate, in both cases, and so [a] is not very good at discriminating the relevant conditions even in such a simple case as this. Nevertheless, Goodman decides that the proper course is to eliminate both counterfactuals and tacks [b] on. But Parry points out in « A Reexamination of the Problem of Counterfactual Conditionals » that [a] is actually very indiscriminating and that, as long as A is compatible with C (*i.e.*, consistent relative to the laws), it will be defeated (let through any counterfactual) by the formal trick of using as S the material conditional $A \Rightarrow C$. But, again, instead of trying to analyse what makes a condition legitimate — and after deflating Parry's attempt to do so in « Parry on Counterfactuals », — Goodman adds the purely *ad hoc* formal condition « that neither S nor $\neg S$ follows by law from $\neg A$. » This doesn't do any good, however, because if you take a true sentence Q which doesn't follow by law from $\neg A$ and which is compatible with A and with C, you defeat [a] again by $(A \Rightarrow C)$ & Q (and similarly for [b]). So, this sort of formal processing has not given us *any idea* of what makes a condition relevant.

Moreover, going back to Jones, suppose that he lives in North Carolina near the Virginia border and works in Virginia. He goes home every night, but one night his car conks out and he has to spend the night in his office. You may correctly argue that if he were in Carolina, he would be in North Carolina. But the split clauses approach is also indiscriminating in its own right and it will cut this out just as it cut out the earlier false counterfactual by letting in « If Jones were in Carolina, he would be in South

Carolina » with the help of illegitimate information. So, Goodman didn't get anywhere through this kind of formal approach.

I'm not arguing that formal stuff is bad, or misleading, or worthless, or anything of the kind, but only that one can't go at it blindly because it *depends* on understanding and it doesn't *substitute* it. (And there are no safe rules of processing for understanding.)

39. « On What There Is », p. 2.

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(Whenever a reprint is indicated, page references are to this reprint.)

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